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DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING

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Advanced Aerodynamics

Homework 1

1. Consider a symmetric airfoil of chord length c, and thickness ratio $\theta = 12\%$ [1] at an angle of attack of $\alpha = 5.7^o$ to a uniform upstream flow V_∞. The velocity along the surface of the airfoil is approximated by

$$V_\pm = V_\infty \left[1 \pm \alpha \sqrt{\frac{1-x}{1+x+1.3\theta^2}} - 0.25\theta \frac{1 \pm 3.58\alpha}{1+x+0.03} \right], \tag{1}$$

where we have non-dimensionalized lengths by $c/2$ and $V_\pm$ is the velocity along the suction side for + and the pressure side for -, respectively. The velocity and airfoil are plotted in figure(1). Calculate the lift and moment coefficients. Find the center of pressure.

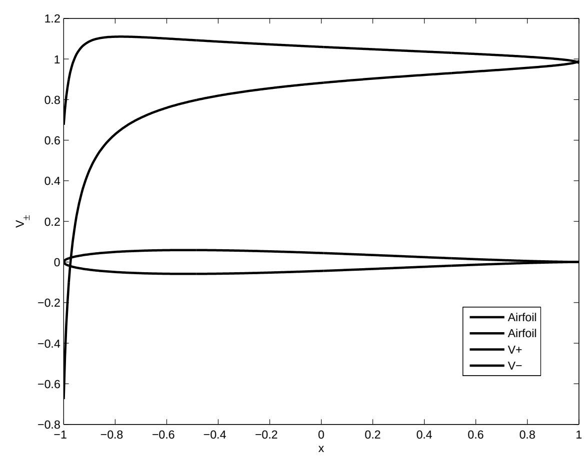

Figure 1: The Airfoil Surface Velocity.

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[1] The thickness ratio is the maximum thickness divided by the chord length.

2. The velocity components for a particular flow field are given by

$$u = 16x^2 + y, \quad (2)$$
$$v = 10, \quad (3)$$
$$w = yz^2. \quad (4)$$

(a) Determine the circulation, Γ, for this flow field around the following contour:

$$\begin{aligned} 0 \leq x \leq 10 &: \quad y = 0, \\ 0 \leq y \leq 5 &: \quad x = 10, \\ 0 \leq x \leq 10 &: \quad y = 5, \\ 0 \leq y \leq 5 &: \quad x = 0. \end{aligned}$$

(b) Calculate the vorticity vector, $\vec{\zeta}$, for the given flow field and evaluate

$$\int_{\Sigma} \vec{\zeta} \cdot \vec{n} d\Sigma,$$

where Σ is the area of the rectangle defined in (a), and $\vec{n}$ is the unit outward normal to the area. Compare the result obtained in (b) with that obtained in (a).

3. The velocity components in cylindrical coordinates for a uniform flow around a circular cylinder are

$$u_r = U(1 - \frac{a^2}{r^2})\, cos\theta, \quad (5)$$
$$u_\theta = -U(1 + \frac{a^2}{r^2})\, sin\theta - \frac{\Gamma}{2\pi r}, \quad (6)$$

where U is the upstream velocity and a is the radius of the cylinder. We assume the fluid density ρ to be constant and viscous effects are negligible. We also neglect body forces. It is helpful to non-dimensionalize length, velocity and pressure with respect to a, U, and $(1/2)\rho U^2$, respectively. It is also convenient to introduce the parameter $\Gamma^* = \Gamma/(4\pi U a)$.

(a) Calculate the vorticity of the velocity field (5, 6). Find the velocity potential if it exists.

(b) Calculate the circulation of the velocity field around any closed circuit surrounding the circle.

(c) Apply Stokes theorem to find the relation between circulation and vorticity and compare with the results of (3a, and 3b). Comments.

(d) Show that you can use Bernoulli (??) to determine the pressure $p(r, \theta)$ at any point in the fluid. Take the pressure far from the cylinder to be constant and equal to p_0.

(e) Calculate and plot the pressure distribution, $p(a, \theta)$ along the surface of the cylinder for $\Gamma^* = 0, 0.5, 1, 2$.

(f) Calculate the force applied on the cylinder by the fluid motion.

(g) Find the location of the stagnation points for $\Gamma^* = 0, 0.5, 1, 2$.